Fiske steps and Abrikosov vortices in Josephson tunnel junctions

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We present a theoretical and experimental study of the Fiske resonances in the current-voltage characteristics of "small" Josephson junctions with randomly distributed misaligned Abrikosov vortices. We obtained that in the presence of Abrikosov vortices, the resonant interaction of electromagnetic waves, excited inside a junction, with the ac Josephson current manifests itself by Fiske steps in a current-voltage characteristics even in the absence of external magnetic field. We found that the voltage positions of the Fiske steps are determined by a junction size but the Fiske-step magnitudes depend both on the density of trapped Abrikosov vortices and on their misalignment parameter. We measured the magnetic field dependence of both the amplitude of the first Fiske step and the Josephson critical current of low-dissipative small Nb-based Josephson tunnel junctions with artificially introduced Abrikosov vortices. A strong decay of the Josephson critical current and a weak non-monotonic decrease in the first Fiske-step amplitude on the Abrikosov vortex density were observed. The experimentally observed dependencies are well described by the developed theory.

I. INTRODUCTION

It is a well known that the current-voltage (I-V) characteristics of Josephson tunnel junctions can display sharp current resonances. These interesting features, called Fiske steps, appear as a result of resonant interaction between ac Josephson current and standing electromagnetic waves that can be excited in the junction.[1] A long-time ago, the Fiske steps have been experimentally observed in planar Josephson tunnel junctions.[2] In this time, the complete theoretical description of the Fiske steps in the case of a junction with low-quality factor has been done by Kulik in Ref. 3. After that the Fiske steps have been observed in numerous Josephson coupled systems, e.g., Josephson-junction arrays,[4] stacked artificial Josephson junctions,[5] intrinsic high-T_c Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ stacked superconducting tunnel junctions,[6] and high-T_c YBa$_2$Cu$_3$O$_{7-\delta}$ bicrystal Josephson junction.[7] Recently, the Fiske steps have been investigated theoretically in 0-π Josephson junctions[8] and they have been experimentally observed in 0-π Josephson tunnel junctions with ferromagnetic barrier.[9]

In "small" Josephson junctions in which a junction size is less than the Josephson penetration depth λ_j, the spectrum of the electromagnetic waves has a form, $\omega(k)=c_0 k$, where c_0 is the Swihart velocity, and the wave vectors k_n are determined by a junction´s size W as $k_n=\pi n/W$ and $n=1,2,\dots$ The voltage positions of nth Fiske steps V_n are determined by the spectrum $\omega(k_n)$ of the electromagnetic waves as[1]

$$V_n = \frac{\hbar\omega(k_n)}{2e}. \qquad (1)$$

The magnitude of Fiske steps depends strongly on the spatial dependence of the Josephson phase difference between the electrodes $\varphi(\vec{\rho})$, where $\vec{\rho}=\{x,y\}$ are the coordinates in the junction plane.

In the absence of inhomogeneities, a simplest way to create a coordinate dependence of the Josephson phase differ-

ence φ is to apply an external magnetic field parallel to the junction plane. In this case, the Josephson critical current is strongly diminished but the magnitude of the Fiske step increases substantially. The magnetic field dependence of Fiske steps has been investigated in Josephson tunnel junctions with different shapes, namely, square-shaped junction,[10] circular-shaped junction,[11] and quatric-shaped junction.[12] In Ref. 13, the Fiske steps have been experimentally and theoretically investigated in the case of the annular-shaped Josephson junction both in an external magnetic field and in a magnetic field generated by the injection current passed along one of the annular junction electrode. However, even in the absence of externally applied magnetic field, the Fiske resonances can appear in *nonuniform Josephson junctions*. Indeed, as was observed in Ref. 14, the randomly distributed barrier inhomogeneities (the structural fluctuations) provide necessary conditions for generation of Fiske steps without external magnetic field.

From the experimental point of view, the nonuniform Josephson junctions can be formed by including artificial inhomogeneities in the tunnel barrier[15] or by introducing Abrikosov vortices.[16,17] Inhomogeneities of the first type suppress locally the Josephson tunneling, and, therefore, the spatial variation in Josephson critical current density occurs. These inhomogeneities may be included in the junction barrier during the fabrication process. So a change of some property of the inhomogeneities (for example, their density) implies necessity to fabricate several Josephson junctions in which this property varies. Obviously, the practical realization of this kind of experiment is a rather complicated task.

On the other hand, the Abrikosov vortices can be easily trapped in the junction by applying an external magnetic field *perpendicular* to the junction plane. Moreover, if pinning centers are randomly distributed in superconducting electrodes, the Abrikosov vortices can be trapped in the particular form of a misaligned vortex such that the magnetic field of the vortex enters and leaves the superconducting electrodes of a junction in different points (see Fig. 1). We
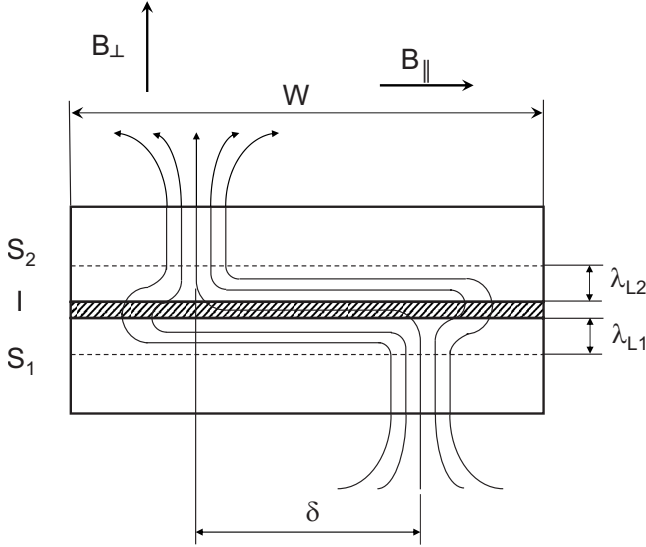
FIG. 1. Schematic cross section of a Josephson tunnel junction with a misaligned Abrikosov vortex. δ is the misalignment length. λ_{L_1} and λ_{L_2} are the London penetration depths of the superconductors S_1 and S_2 separated by insulating layer I forming the tunnel barrier. The orientation of the two magnetic field, $B_\perp$ and $B_\parallel$, is provided.

notice here that an extreme case of misaligned vortices, so-called "pancakes" vortices, naturally appears in high-T_c superconductors.[18] The misaligned Abrikosov vortices can be considered as local magnetic inhomogeneities leading to an additional spatial variation in the Josephson phase difference $\varphi(\vec{\rho})$, and, hence, the variation in the Josephson critical current density over a junction area.[16,17,19,20] By means of field-cooling process, one can change the density of Abrikosov vortices in the same junction. The possibility to trap Abrikosov vortices in a relatively easy way offers many opportunities for experimental studies of Josephson junctions with inhomogeneities. Thus, the dependence of Josephson critical current on the Abrikosov vortices density and an externally applied magnetic field have been intensively studied[16,17,19–22] many years ago. Recently, in Refs. 23 and 24, a Josephson junction has been used as a tool for monitoring of the position of a single Abrikosov vortex in thermal depinning and Lorentz-force depinning experiments. The recent experiments on a spontaneous fluxon formation in annular Josephson junctions demonstrate the necessity to take into account the Abrikosov vortices trapping during the thermal quench as a competitive effect.[25] In addition, a new detection principle based on the interaction of a single gamma-ray photon with trapped Abrikosov vortex is proposed for the development of a gamma-ray solid-state detector with high intrinsic detection efficiency in the energy range up to 100 keV.[26]

In this paper, we present a theoretical study of the Fiske steps for a small Josephson junction in the presence of randomly distributed Abrikosov vortices. Our approach is based on the extension of a well-known Kulik analysis for low-dissipative uniform Josephson junctions[27] to the Josephson junctions with inhomogeneities. We obtain a peculiar regime where the Fiske steps amplitude shows a weak nonmonotonic decrease with the vortex density n_A. We experimentally

trapped different number of Abrikosov vortices and for each vortex density n_A, we measured the dependencies of the critical current I_c and the first Fiske step amplitude $I_F^{(1)}$ on n_A and on externally applied parallel to the junction plane magnetic field $B_\parallel$. A good agreement between our theory and experimental results was found.

The paper is organized as follows. In Sec. II, the theoretical model of a Josephson junction with randomly distributed misaligned Abrikosov vortices is presented. In Sec. III, by making use of a generic approach elaborated in the Ref. 27, we calculate the Fiske resonances in inhomogeneous Josephson junctions, and in the Sec. IV, the dependence of the Fiske steps amplitude $I_F^{(n)}$ on the density of Abrikosov vortices n_A is analyzed. Section V is dedicated to the experimental details, namely, the experimental setup, the sample description and the procedure to measure the Josephson critical current I_c and the amplitude of first Fiske step $I_F^{(1)}$. In Sec. VI, we present the experimental results and the comparison with the theory. Section VII provides conclusions.

II. MODEL OF A JOSEPHSON JUNCTION WITH RANDOMLY DISTRIBUTED MISALIGNED ABRIKOSOV VORTICES

We consider a small, i.e., $W < \lambda_J$, Josephson junction in the presence of randomly distributed pinned misaligned Abrikosov vortices (here, W is the size of a square Josephson junction and λ_J is the Josephson penetration depth). A magnetic field of a misaligned Abrikosov vortex enters and leaves the superconducting electrodes of a junction in different points. The distance between these points determines the misalignment length δ (see Fig. 1).

Under these conditions, the Abrikosov vortex contribution $\varphi_V(\vec{\rho})$ to the total Josephson phase difference $\varphi(\vec{\rho}, t)$ randomly depends on the position $\vec{\rho}$ in the plane of the junction. The particular form of a single Abrikosov vortex contribution has been obtained previously in Refs. 19 and 20.

The dynamics of a Josephson junction with randomly distributed Abrikosov vortices is described by an inhomogeneous sine-Gordon equation,[1,28,29]

$$\frac{\partial^2 \varphi_1(\vec{\rho},t)}{\partial \vec{\rho}^2} - \frac{1}{c_0^2} \frac{\partial^2 \varphi_1(\vec{\rho},t)}{\partial t^2} - \frac{\gamma}{c_0^2} \frac{\partial \varphi_1(\vec{\rho},t)}{\partial t} = \lambda_J^{-2} \sin[\varphi_V(\vec{\rho}) + k_0 x$$
$$+ \omega_0 t + \varphi_1(\vec{\rho},t)], \qquad (2)$$

where

$$k_0 = \frac{2\pi B_\parallel (\lambda_{L1} + \lambda_{L2})}{\Phi_0}, \quad \omega_0 = \frac{2eV}{\hbar}.$$

Here V is the external dc voltage, c_0 is the velocity of Swihart waves, $B_\parallel$ is an external magnetic field applied parallel to the y axis, λ_{L_1} and λ_{L_2} are the London penetration depths of the superconductors forming the junction, the parameter γ determines the decay of the electromagnetic waves in the junction and depends on the quasiparticle resistance of the junction in a subgap region, and Φ_0 is the flux quantum.

III. FISKE RESONANCES IN INHOMOGENEOUS JOSEPHSON TUNNEL JUNCTIONS: THEORY

Our theoretical analysis is based on the extension of the Kulik approach developed for small low-dissipative Josephson junctions with $\gamma \ll \omega_0$.[1,27] Thus, we present $\varphi_1(\vec{\rho}, t)$ in the following form:

$$\varphi_1(\vec{\rho}, t) = \sum_n \Re e\{a_n e^{-ik_n x - i\omega_0 t}\}, \tag{3}$$

where the wave vectors of electromagnetic waves that can be excited in the junction, i.e., $k_n = \pi n / W$, are determined by the junction's geometry. We note that in the particular case when the parameter γ is not so small, Fiske resonances overlap each other, and a single Eck peak is formed in the I-V characteristic of a Josephson junction.[1] Such feature has been theoretically analyzed also for Josephson junctions with randomly distributed inhomogeneities.[28]

As the parameter $\gamma \ll \omega_0$, the Fiske resonances are well separated, and for the analysis of nth Fiske step, we can choose a single wave vector k_n in the Eq. (3). Substituting Eq. (3) in Eq. (2) and using the condition $\hbar\omega_0 = c_0 k_n$, i.e., (1), we obtain the nonlinear equation for the amplitude of Swihart electromagnetic waves as

$$a_n = \frac{c_0^2}{\lambda_J^2 \gamma \omega_0} \left[J_0^2\left(\frac{a_n}{2}\right) + J_1^2\left(\frac{a_n}{2}\right) \right] \Re e\left[\int \frac{d^2\vec{\rho}}{S} e^{i\varphi_V(\vec{\rho}) + ik_0 x - ik_n x} \right], \tag{4}$$

where J_0 and J_1 are the Bessel functions and S is the junction area.

Similarly we obtain the expression for the amplitude of nth Fiske step,

$$I_F^{(n)} = j_c J_0\left(\frac{a_n}{2}\right) J_1\left(\frac{a_n}{2}\right) \Re e\left[\int d^2\vec{\rho} \, e^{i\varphi_V(\vec{\rho}) + ik_0 x - ik_n x} \right], \tag{5}$$

where j_c is the Josephson critical current density in the absence of vortices.

IV. CRITICAL CURRENT AND FISKE RESONANCES: THE DEPENDENCE ON THE ABRIKOSOV VORTICES DENSITY

Since the contribution of Abrikosov vortices $\varphi_V(\vec{\rho})$ to the Josephson phase difference is a random function of the coordinate $\vec{\rho}$, we obtain the averaged quantities only. First, we define the correlation area for Abrikosov vortices as

$$\sigma(k) = \left\langle \int d^2\vec{\rho} \, e^{i[\varphi_V(\vec{\rho}) - \varphi_V(0)] + ikx} \right\rangle, \tag{6}$$

where the sign $\langle \cdots \rangle$ means the averaged value over the random positions of Abrikosov vortices. The correlation area $\sigma(k)$ depends on the density of misaligned Abrikosov vortices n_A, the misalignment length δ, and the wave vector k that, in turn, is determined by an externally applied magnetic field parallel to the junction plane $B_\parallel$. For the particular case of a circular junction, the $\sigma(k)$ has been found in Refs. 16, 17, and 20 and in the absence of $B_\parallel$,

$$\sigma \simeq S \exp[-\pi n_A \delta^2 \ln(2L/\delta)], \tag{7}$$

where L is the radius of the circular junction. It has been obtained in Refs. 16, 17, and 20 that the critical current of the Josephson junction with randomly distributed misaligned Abrikosov vortices is determined by the correlation area $\sigma(k)$ as

$$I_c = j_c \sqrt{\sigma(k)S}. \tag{8}$$

Thus, in the absence of $B_\parallel$ as the critical current reaches the maximum, we obtain a strong decrease in the critical current I_c with the Abrikosov vortices density n_A,

$$\ln\frac{I_c^{max}}{I_{c0}} = \frac{n_A \pi \delta^2}{2} \ln\frac{\delta}{2L}, \tag{9}$$

where I_{c0} is the Josephson critical current in the absence of Abrikosov vortices.

Next, we turn to the analysis of Eqs. (4) and (5) determining the amplitudes of Fiske resonances. First, we notice that even in the absence of k_0 (or $B_\parallel$), the amplitudes of Fiske resonances $I_F^{(n)}$ are not zero. However, the magnitudes of Fiske steps can be increased by tuning of $B_\parallel$. The maximum values of $I_F^{(n)}$ reach as the condition $k_n = k_0$ is satisfied. In the following, we will assume that the correlation area σ is small, i.e., $\sigma \ll S$. Using this assumption, we express the maximum amplitudes of Fiske resonances $I_F^{(n)}$ through a single parameter $\sigma(0) = \sigma$. Indeed, in the limit as σ is small or more precisely $\sigma \ll S(\frac{\lambda_J^2 \gamma \omega_0}{c_0^2})^2 = \sigma^*$, the amplitudes a_n are small, and expanding the Bessel function over a small argument a_n, we obtain the amplitude of the first Fiske step

$$I_F^{(1)} = \frac{j_c \sigma}{4} \sqrt{\frac{S}{\sigma^*}} = j_c \sigma c_0^2 (4\lambda_J^2 \gamma \omega_0)^{-1}, \quad \sigma \ll \sigma^*. \tag{10}$$

In this regime, $I_F^{(1)}$ is proportional to σ and displays a strong exponential decrease with n_A.

In the opposite limit $\sigma \gtrsim \sigma^*$, the amplitude of electromagnetic waves becomes large but the oscillations of Bessel functions are strongly suppressed. Therefore, the amplitude of $I_F^{(1)}$ is still small. The averaged value of $I_F^{(1)}$ can be also expressed through a single parameter σ as

$$I_F^{(1)} = \left(\frac{32}{\pi}\right)^{1/3} j_c S\left(\frac{\sigma^2 \sigma^*}{S^3}\right)^{1/6} e^{-3/8[4\sigma/\pi\sigma^*]^{1/3}}, \quad \sigma \gg \sigma^*. \tag{11}$$

In this limit, the amplitude of Fiske resonance weakly depends on the density of Abrikosov vortices, displaying a small maximum at $\sigma \simeq \sigma^*$. Notice here, that this regime can be easily realized for low-dissipative junctions as $\gamma \ll \omega_0$.

V. JOSEPHSON JUNCTIONS WITH MISALIGNED ABRIKOSOV VORTICES: EXPERIMENTAL SETUP AND MEASUREMENTS

The experiments were carried out on the Nb-AlO$_x$-Nb Josephson tunnel junction of square geometry and dimensions of 50×50 μm^2. Details of the fabrication process are reported in Refs. 29 and 30.
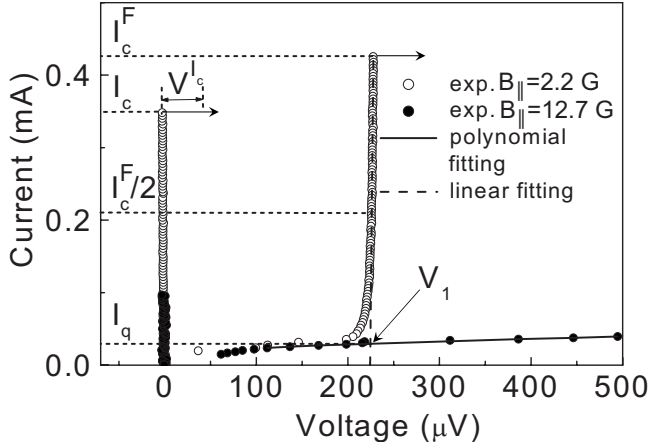
FIG. 2. *I-V* curves recorded at $B_\parallel = 2.2$ G when the first Fiske step is maximized (open circles) and at $B_\parallel = 12.7$ G when the Fiske step is suppressed (closed circles). The method allowing to extract both the amplitude of the first Fiske step, $I_F^{(1)}$, and its voltage position, V_1, is shown. The experimental first Fiske-step branch is fitted by a linear function in the current interval between $I_c^F/2$ and I_c^F (dashed line). The experimental quasiparticle branch is fitted by a polynomial function (solid line). The voltage position V_1 is obtained from the point of the intersection of the dashed line with the solid line. The amplitude of the first Fiske step is $I_F^{(1)} = I_c^F - I_q$, where I_q is the quasiparticle current at voltage V_1. V^{I_c} is the threshold voltage to define the Josephson critical current.

The junction was mounted on the sample holder inside a closed copper box in order to prevent the influence of the electromagnetic noise. The box was immersed in a liquid-helium bath so all measurements were done at a stable temperature $T = 4.2$ K. In order to heat the junction above the superconducting transition temperature T_c, a SMD 100 Ω resistor was used as a heater mounted in contact with the sample holder. A special coil system containing two Helmholtz copper coils was used in order to generate both the magnetic fields perpendicular to the junction plane $B_\perp$ and the magnetic field parallel to the junction plane $B_\parallel$. The shielding of the sample from the Earth magnetic field and from the electromagnetic noise was reached by using a three-level μ-metal shielding system and a one-level Al shield.

The accurate measurements both the Josephson critical current I_c, and the amplitude of the first Fiske step $I_F^{(1)}$ were achieved by a careful shielding and by inserting cold low-pass filter, mounted just near the sample. First, we measured the *I-V* curves of the junction without trapped Abrikosov vortices. We tuned the amplitudes of Fiske resonances by an externally applied magnetic field $B_\parallel$. Figure 2 shows the typical *I-V* curves where the amplitude of the first Fiske step was maximized (open circles) and suppressed (closed circles). Both *I-V* curves were used in order to obtain the voltage position of the first Fiske step V_1 and the quasiparticle current at the Fiske-step voltage I_q.

The method to determine the voltage position V_1 of the first Fiske step is shown in Fig. 2. We note that the voltage position can also be estimated as the voltage at current I_c^F (see Fig. 2) but the difference between this method and the one represented in Fig. 2 is negligible.

The experimental technique to measure the Josephson critical current I_c was as follows. The current I was increased

linearly in time from zero value, and the junction voltage V was simultaneously recorded. When the current reached the Josephson critical current value, a nonzero junction voltage appeared. Under a condition that the junction voltage equals to the threshold voltage V^{I_c} (criterion of the Josephson critical current, see Fig. 2), the current value was recorded and the current returned back to zero and then the next measuring cycle started again. The current sweep frequency was 10 Hz and the value of V^{I_c} was of 10 μV.

A cycle to measure the amplitude of the first Fiske step was similar to the Josephson critical current measurements but the starting value of the current was chosen to satisfy the condition that the junction voltage was greater then zero and less then voltage position of the first Fiske step. From this starting point, the current was increased, the first Fiske-step branch was recorded and the maximum current at the first Fiske-step branch I_c^F was defined as the current at which the junction voltage became greater then $V_1 + V^{I_c}$ (see Fig. 2). The amplitude of the first Fiske step $I_F^{(1)}$ was determined by substracting the quasiparticle current at Fiske-step voltage I_q from the switching current at Fiske branch I_c^F. Both Josephson critical current and amplitude of the first Fiske step for a given value of parallel magnetic field $B_\parallel$ were obtained. The $I_c(B_\parallel)$ and $I_F^{(1)}(B_\parallel)$ dependencies were measured for both polarities of the bias current.

The experimental cycle for the evaluation of the influence of Abrikosov vortices on the first Fiske-step amplitude consisted of the following steps: (1) the sample was heated to $T > T_c$; (2) a magnetic field $B_\perp$ of fixed value was applied; (3) the junction was cooled in the field $B_\perp$ up to $T = 4.2$ K, i.e., immersed in the liquid helium at normal pressure; (4) the magnetic field $B_\perp$ was turned off; (5) the amplitude of the first Fiske step was maximized by the parallel magnetic field $B_\parallel$ and the *I-V* curve with the first Fiske-step branch was measured in order to estimate the voltage position of the first Fiske step; (6) the $I_c(B_\parallel)$ curve was measured; (7) the dependence of the maximum current at the first Fiske-step branch I_c^F on the parallel magnetic field $B_\parallel$ was measured; (8) both the Josephson critical current and the amplitude of the first Fiske step were suppressed by a suitable parallel magnetic field $B_\parallel$ and the *I-V* curve was recorded in order to measure precisely the voltage position of the first Fiske step V_1 and the quasiparticle current at $V = V_1$; (9) the dependence of the first Fiske-step amplitude $I_F^{(1)}$ on parallel magnetic field $B_\parallel$ was obtained from $I_c^F(B_\parallel)$ curve by extracting I_q from I_c^F.

After the field-cooling process, the Abrikosov vortex density n_A can be estimated as

$$n_A \simeq \frac{B_\perp}{\Phi_0}. \tag{12}$$

VI. EXPERIMENTAL RESULTS AND DISCUSSION

A. Junction parameters

The measurements of the $I_c(B_\parallel)$ dependence without trapped Abrikosov vortices were performed at $T = 4.2$ K (Fig. 3, open circles). The Josephson penetration depth λ_j was estimated to be about 54 μm. The experimental $I_c(B_\parallel)$
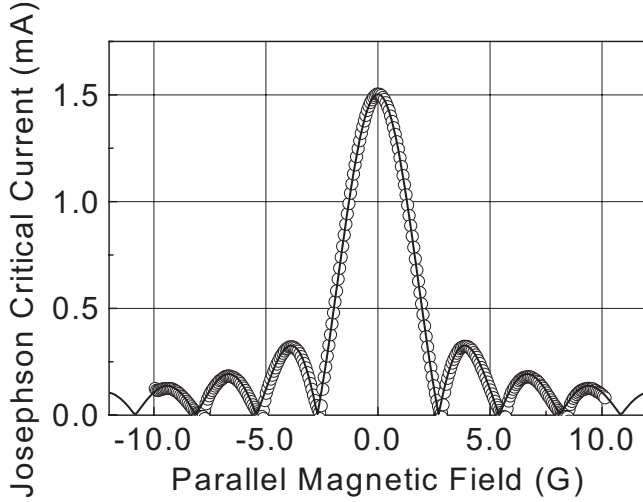
FIG. 3. The experimental $I_c(B_\parallel)$ curve (open circles) and theoretical fit by Eq. (10) (solid line). The trapped Abrikosov vortices are absent.

curve is in a good agreement with the theoretical one which was calculated for a small square junction by the Fraunhofer formula,[1]

$$I_c(B_\parallel) = j_c W^2 \left| \frac{\sin\left(\frac{\pi B_\parallel W d}{\Phi_0}\right)}{\frac{\pi B_\parallel W d}{\Phi_0}} \right|, \qquad (13)$$

where j_c is the Josephson critical current density, W is the dimension of the junction, $d = \lambda_{L_1} + \lambda_{L_2} + t$, λ_{L_1} and λ_{L_2} are the effective London penetration depths of the superconductors forming the junction, and t is the thickness of the insulating layer. The theoretical curve calculated by Eq. (13) is reported in Fig. 3 by solid line. This good agreement between experimental and theoretical results confirms the uniform distribution of the Josephson current density over the junction area.

B. Magnetic field dependence of the first Fiske step without Abrikosov vortices

In the absence of trapped Abrikosov vortices, the value of voltage position of the first Fiske step V_1 was determined from the I-V curve at $T = 4.2$ K when the Fiske-step amplitude was maximized by magnetic field $B_\parallel$ (Fig. 2, open circles). Using this value of V_1 and Eq. (1), we calculated the resonant frequency ω_0 and Swihart velocity c_0. The values of V_1, ω_0, and c_0 are given in Table I.

TABLE I. Experimental values of the voltage position of the first Fiske step V_1, the Swihart velocity, c_0, and the resonant frequency, ω_0, obtained from Eq. (1).

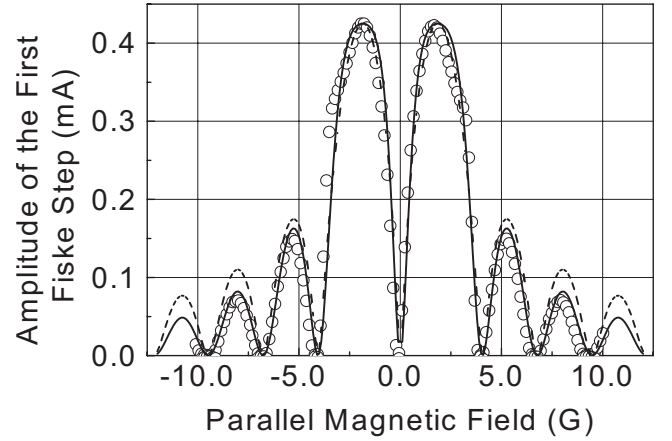| V_1 (μV)	c_0 (m/s)	ω_0 (s^{-1})
226.3	1.1×10^7	6.9×10^{11}

FIG. 4. Experimental $I_F^{(1)}(B_\parallel)$ curve for the junction without trapped Abrikosov (open circles). Solid line represents the theoretical dependence calculated by Eqs. (4) and (5) for $\gamma = 1.1 \times 10^{10}$ s^{-1}. Dashed line is the result of calculation without $J_1^2(\frac{a}{2})$ term at $\gamma = 4.3 \times 10^9$ s^{-1} according to the original theoretical analysis of Ref. 27.

The $I_F^{(1)}(B_\parallel)$ curve measured after cooling the junction in a zero magnetic field $B_\perp$ (open circles) is shown in Fig. 4. The theoretical dependence for the first Fiske-step amplitude on field $B_\parallel$ was numerically computed from Eqs. (4) and (5) in the absence of Abrikosov vortices. The value of the parameter $\gamma = 1.1 \times 10^{10}$ s^{-1} provides a best fitting of experimental data. Similar value of γ can be extracted from the I-V curves where the Fiske resonances were suppressed. The agreement with the theoretical analysis is excellent even for the second and third lobes. In Ref. 27, a similar theoretical analysis of the Fiske resonances of a junction without Abrikosov vortices has been carried out. However, in the equation determining the amplitude a of electromagnetic waves, the term of J_1 was omitted. Taking into account this term, one can get a better agreement with experimental data.

C. Josephson critical current of a junction with trapped Abrikosov vortices

Figure 5 shows $I_c(B_\parallel)$ curves measured after cooling of the sample in different perpendicular magnetic fields $B_\perp$. We obtained from each $I_c(B_\parallel)$ curve the maximum values of I_c for both polarities, I_c^{+max} and I_c^{-max}, and then we selected the maximum value between I_c^{+max} and I_c^{-max} denoting it as I_c^{max}. Similar to Refs. 16 and 17, we found the deviation of the $I_c(B_\parallel)$ curve from a "vortex-free" Fraunhofer-type curve (Fig. 3, open circles) and we observed a strong suppression of I_c^{max}. For each value of $B_\perp$, the vortex density n_A was calculated by using Eq. (12). The dependence of the maximum Josephson critical current I_c^{max} on the density of trapped Abrikosov vortices n_A is shown in Fig. 7 by open squares. It is well known that during the field-cooling process, misaligned Abrikosov vortices are randomly trapped over the junction area (see Fig. 1). Equation (9) describes the dependence of I_c^{max} both on the vortex density n_A and on the misalignment length δ in case of a circular junction. Taking L to be the radius of a circular junction with an area equal
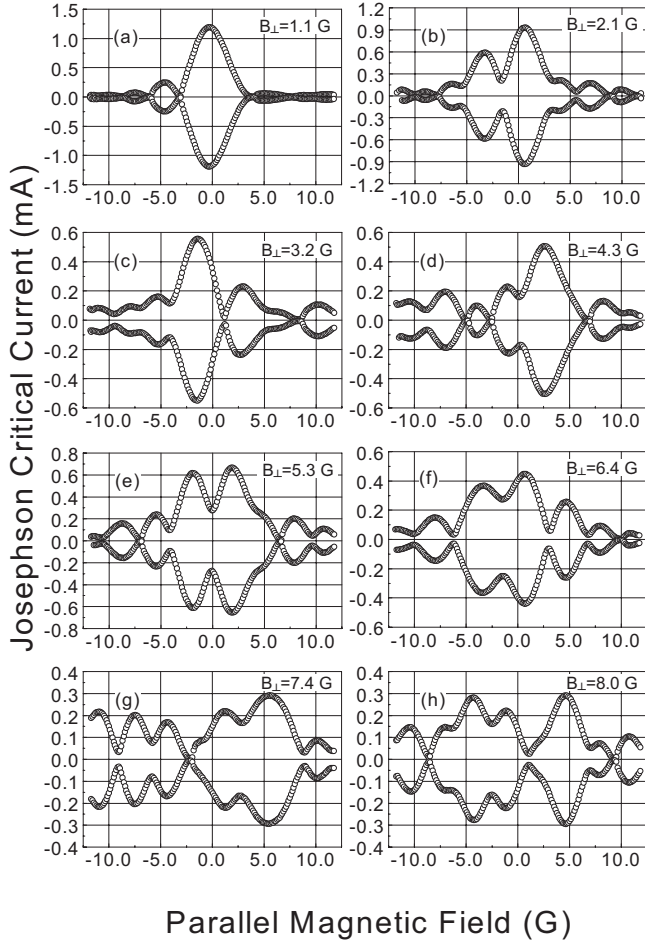
FIG. 5. $I_c(B_\parallel)$ curves measured after cooling of the junction in perpendicular magnetic field $B_\perp$ of various values.

FIG. 6. $I_F^{(1)}(B_\parallel)$ curves measured after the junction was cooled in perpendicular magnetic field $B_\perp$ of various values.

50×50 μm^2, we used Eqs. (9) and (12) in order to extract the value of δ for each value of vortex density. The average value of misalignment parameter over the all vortex densities is $\bar{\delta} = 0.8 \pm 0.1$ μm.

D. First Fiske step in the presence of trapped Abrikosov vortices

We observed that the presence of trapped Abrikosov vortices leads to a great deviation of the $I_F^{(1)}(B_\parallel)$ curves from the vortex-free one shown in Fig. 4 by open circles. These curves are shown in Fig. 6. The presence of trapped vortices did not lead to the variation in the voltage positions of Fiske resonance. This result indicates that the trapped Abrikosov vortices did not change the geometrical conditions for appearance of the first Fiske step and only influenced on the magnitude of $I_F^{(1)}$ by additional spatial variation in the Josephson phase difference $\varphi(\vec{\rho}, t)$.

Next, we notice that a nonzero amplitude of the first Fiske step at $B_\parallel = 0$ appeared as Abrikosov vortices were trapped. However, by tuning the parallel magnetic field we could substantially increase the amplitude of the first Fiske step. Using the same procedure which was used to obtain the I_c^{max} from the $I_c(B_\parallel)$ curves (see Sec. VI C), we found the maximum
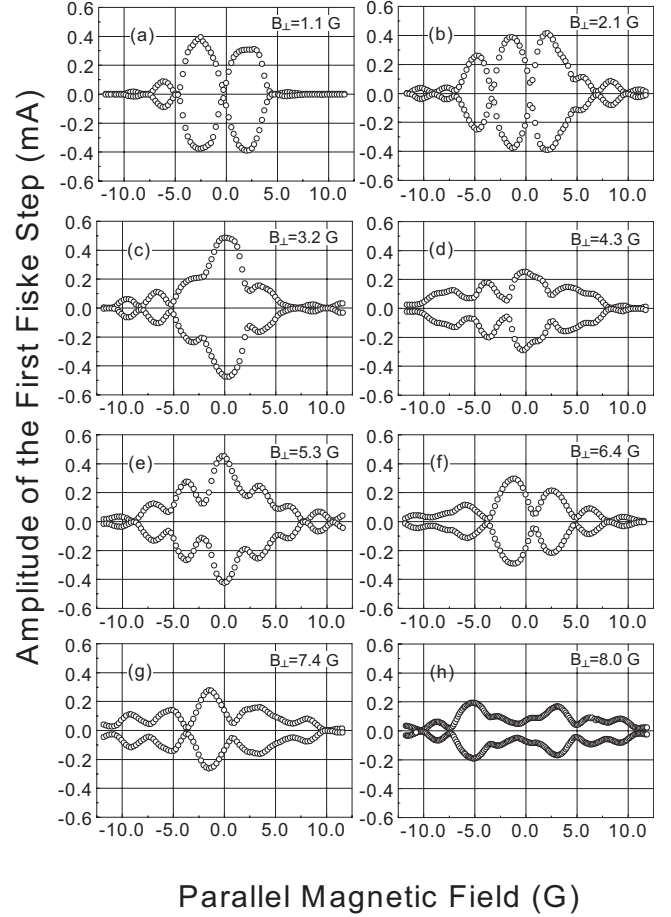
amplitude of the first Fiske step $I_F^{(1),max}$ for each value of $B_\perp$. The experimental dependence of $I_F^{(1),max}$ on the Abrikosov vortex density n_A is shown in Fig. 7 (open circles). In contrast to the $I_c^{max}(n_A)$ dependence, the weak nonmonotonic decrease in the magnitude of $I_F^{(1),max}$ with n_A was observed. Such behavior of the $I_F^{(1),max}(n_A)$ dependence indicates that the condition of $\sigma \geq \sigma^*$ should be applied for our sample. Indeed, this conclusion is confirmed by estimating the values of both σ and σ^* for our junction.

In order to estimate the critical value of σ^*, we have to obtain the dissipation parameter γ. This parameter determines the decay of the electromagnetic waves in the junction, and it can be obtained as $\gamma \simeq (R_q C)^{-1}$, where C is the capacitance of a junction per unit area and R_q is the quasiparticle tunneling junction resistance at voltage $V = V_1$ per unit area.[1] Trapped Abrikosov vortices increase the quasiparticle current by virtue of their normal cores[19,31] and, the value of γ has to also increase. Therefore, the value of $\gamma = 1.1 \times 10^{10}$ s^{-1}, obtained for the junction in the absence of Abrikosov vortices, was not convenient for the case of trapped Abrikosov vortices. We used γ as a fitting parameter in order to reach the good agreement between the experimental value of $I_F^{(1)}$ recorded at $B_\perp = 1.1$ G and the theoretical value calculated by using Eqs. (4) and (5) for the same magnetic field. We obtain $\gamma = 2.2 \times 10^{10}$ s^{-1} from this fitting procedure and the parameter $\sigma^* = 3.2 \times 10^2$ μm^2 was estimated.
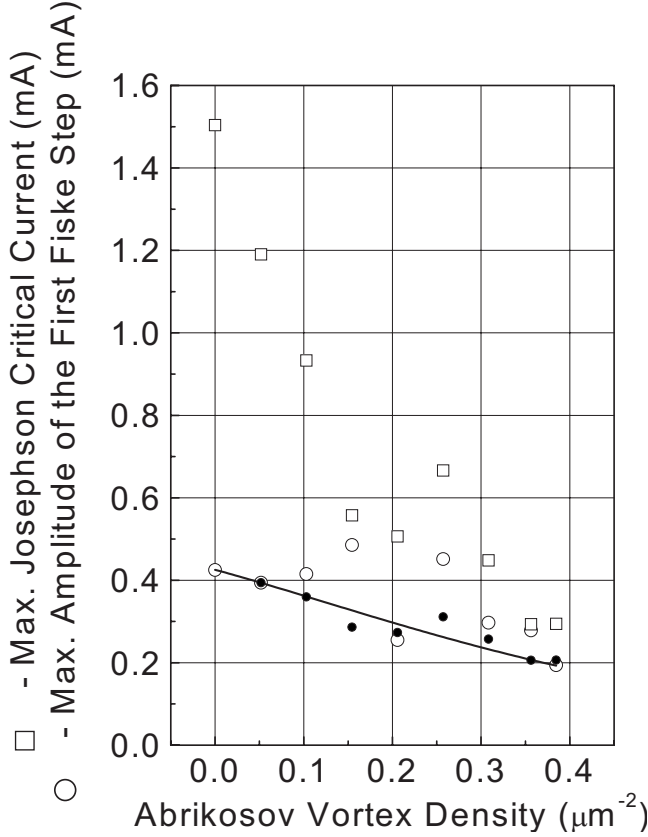
FIG. 7. Open squares: experimental dependence of the maximum Josephson critical current I_c^{max} on the density of trapped Abrikosov vortices n_A; open circles: experimental dependence of the maximum amplitude of the first Fiske step $I_F^{(1),max}$ on the density of trapped Abrikosov vortices n_A. Solid line is the theoretical $I_F^{(1),max}(n_A)$ curve, calculated by Eqs. (7) and (11). The values of $\gamma = 2.2 \times 10^{10}$ s^{-1} and a mean misalignment parameter over all vortex densities $\bar{\delta} = 0.8$ μm were used; full circles represent the theoretical $I_F^{(1),max}(n_A)$ dependence obtained by Eqs. (8) and (11).

It is important to note that the quality factor $Q = \omega_0/\gamma$ of the Fiske resonance is equal to 31, such that a crucial assumption of low-dissipative junctions is valid for our junctions.

Using Eq. (7), we calculated the correlation area σ for each value of density n_A, and we obtained that for all values of $B_\perp$, the correlation area $\sigma \geq \sigma^*$, and therefore, the Eq. (11) is valid. The theoretical dependence of $I_F^{(1),max}$ on n_A is shown in Fig. 7 by solid line.

The experimental dependence $I_F^{(1),max}(n_A)$ agrees qualitatively with the behavior of theoretical curve but several experimental points disagree with our theoretical predictions. One of possible reason of this discrepancy is the variation in the misalignment length δ for different magnetic fields $B_\perp$. In order to check this hypothesis, we extracted σ from the experimental values of I_c^{max} by using Eq. (8) for different values of n_A. Substituting the obtained values of σ into Eq. (11), we calculated $I_F^{(1),max}$ for each experimental values of n_A. The result of such procedure is shown in Fig. 7 by closed circles. By making use of this procedure the agreement between the theoretical dependence and experiment becomes better. So we can conclude that the variation in δ strongly influences the maximum amplitude of the first Fiske step.

However, we have to notice that our theoretical analysis is devoted to *averaged quantity* only. As has been shown in Refs. 20 and 21, in inhomogeneous Josephson junctions strong mesoscopic fluctuations of various physical quantities occur. These fluctuations manifest themselves in the form of random oscillations on the dependence of $I_c^{max}(n_A)$ [or $I_c(B_\parallel)$]. These oscillations can be especially large for the dependence $I_F^{(1),max}$ on n_A due to a strong nonlinearity of Eqs. (4) and (5).

The pronounced deviation of the experimental point at $n_A = 0.15$ μm^{-2} ($B_\perp = 3.2$ G) and $n_A = 0.26$ μm^{-2} ($B_\perp = 5.3$ G) from the theoretical prediction (see Fig. 7) can be also attributed to the trapping of so-called "monopole" Abrikosov vortex. The monopole vortex penetrates only a one superconducting electrode of the tunnel junction and its magnetic field spreads over the total area of a small Josephson junction. Such Abrikosov vortex trapped near the junction center, alters the $I_c(B_\parallel)$ dependence from the Fraunhofer pattern to the one with two primary lobes and with suppressed Josephson critical current at $B_\parallel = 0$. In addition, the amplitude of the first Fiske step is maximized. Even if a large number of Abrikosov vortices is trapped, the effect of the monopole vortex, pinned in the central part of junction area, dominates.[32] Because of a statistical character of the trapping process, the cooling in the fields $B_\perp = 3.2$ G and $B_\perp = 5.3$ G can provide the trapping of monopole Abrikosov vortex near the junction center, which, in turn, maximize the first Fiske step, similar to the magnetic flux trapping effect in case of a Josephson junction with annular geometry.[33] The shapes of the $I_c(B_\parallel)$ dependencies shown in Figs. 5(c) and 5(e) (especially the curve e), permit to sustain our explanation of the deviation of the experimental data from theoretical prediction presented in Fig. 7.

VII. CONCLUSIONS

We have developed a theoretical and experimental study of the Fiske-step resonances in low-dissipative Josephson junctions with randomly distributed misaligned Abrikosov vortices. This analysis is an extension of the approach elaborated in Ref. 27 to inhomogeneous Josephson junctions. The local inhomogeneous magnetic field of misaligned Abrikosov vortices leads to an additional contribution to the Josephson phase difference. The influence of randomly distributed misaligned Abrikosov vortices was described in terms of the correlation area σ [see, Eq. (6)] which depends both on the Abrikosov vortex density n_A and on the misalignment length of vortices δ. We obtained a peculiar regime as $\sigma \geq \sigma^* = S\left(\frac{\lambda_J^2 \gamma \omega_0}{c_0^2}\right)^2$ characterized by a weak nonmonotonic dependence of the maximum Fiske-step amplitude on n_A. In this regime, the amplitude of excited electromagnetic waves in the junction $a_n \geq 1$. This behavior is in a sharp contrast with a strong decrease in the maximum critical current I_c^{max} with n_A. The critical parameter σ^* depends on the dissipation parameter γ of the junction. For low-dissipative junctions, the parameter $\sigma^* \ll S$.

We carried out the experiments on small low-dissipative Nb-based Josephson tunnel junction with artificially introduced Abrikosov vortices. We measured both the critical cur-

rent I_c and the Fiske resonant steps in the I-V curves. We observed that the critical current I_c strongly decreases with the density of Abrikosov vortices in a complete agreement with a theory, Eq. (8).

In the absence of trapped Abrikosov vortices, the experimental dependence of the amplitude of first Fiske resonance on $B_\parallel$ is in a good agreement with the theoretical description by Eqs. (4) and (5), where the term J_1 is taken into account. As the misaligned Abrikosov vortices were introduced in the junction, Fiske resonances were observed *even* in the absence of $B_\parallel$. However, an application of a small magnetic field $B_\parallel$ allowed to increase the amplitude of Fiske resonances. The voltage position of the first Fiske resonance determined by a geometry of the junction, did not vary in the presence of Abrikosov vortices. The measured dependence of the maximum amplitude of first Fiske step $I_F^{(1),max}$ on n_A showed the qualitative agreement with our theoretical analysis [Eq. (11)], i.e., with a weak nonmonotonic dependence (see Fig. 7). Therefore, the regime $\sigma \geq \sigma^*$ for our low-dissipative junctions was realized.

Finally, we notice that Eq. (11) describes the *averaged*

value of the amplitude of Fiske resonances. However, as was indicated in Refs. 20 and 21, strong mesoscopic fluctuations of various physical values can be observed in inhomogeneous Josephson junctions. These mesoscopic fluctuations are especially strong for the amplitude of Fiske resonances due to nonlinear character of Eqs. (4) and (5). Therefore, observed deviations of some experimental point from the theoretical curve described by Eq. (11), can be attributed to such mesoscopic fluctuations. This discrepancy could be also due to a possible trapping of a single *monopole* Abrikosov vortex just near the junction center.

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